Maximizing the Profit of a Farmer

Math 1010 Intermediate Algebra Group Project

In this project your group will solve the following situation:

A farmer has 10 acres total which he can plant in wheat and rye. He has to plant at least 7 acres. However, he has only $1200 to spend and each acre of wheat costs $200 to plant and each acre of rye costs $100 to plant. Moreover, the farmer has to get the planting done in 12 hours and it takes an hour to plant an acre of wheat and 2 hours to plant an acre of rye. If the profit is $500 per acre of wheat and $300 per acre of rye how many acres of each should be planted to maximize profits? Let x be the number of acres of wheat he plants and y be number of acres of rye he plants.

1. Write down a two linear inequalities for the number of acres he can plant. Hint: the number of acres of wheat plus the number of acres of rye has to be at least 7 (first inequality) but no more than his total acreage of 10 (second inequality).

$$x+y \geq 7$$
$$x+y \leq 10$$

2. Write down a linear inequality for the cost to plant the crops.

$$200x + 100y \leq 1200$$

3. Write down a linear inequality for the time to plant the crops.

$$x+2y \leq 12$$

There are two other linear inequalities that must be met. These relate to the fact that the farmer cannot plant a negative numbers of acres. These inequalities are as follows:

$$x \geq 0$$
$$y \geq 0$$

4. Next, write down the profit function for the sale of x acres of wheat and y acres of rye:

$$P = 500x + 300y$$

You now have five linear inequalities and a profit function. These together describe the farmers planting situation. These together make up what is known mathematically as a **linear programming** problem. Write all of the inequalities and the profit function together below. This is typically written one on top of another, with the profit function last.

A $x+y \geq 7$

B $x+y \leq 10$

C $200x+100y \leq 1200$

D $x+2y \leq 12$

E $P = 500x+300y$

$\frac{100y}{100} = \frac{500x+P}{300}$

$y \leq \frac{5}{3}x+P$

$200(0)+100(10)$

$200x+100y \leq 1200$

$-200x \quad -200x$

$\frac{100y}{100} \leq \frac{1200-200x}{100}$

$y \leq 12-2x$

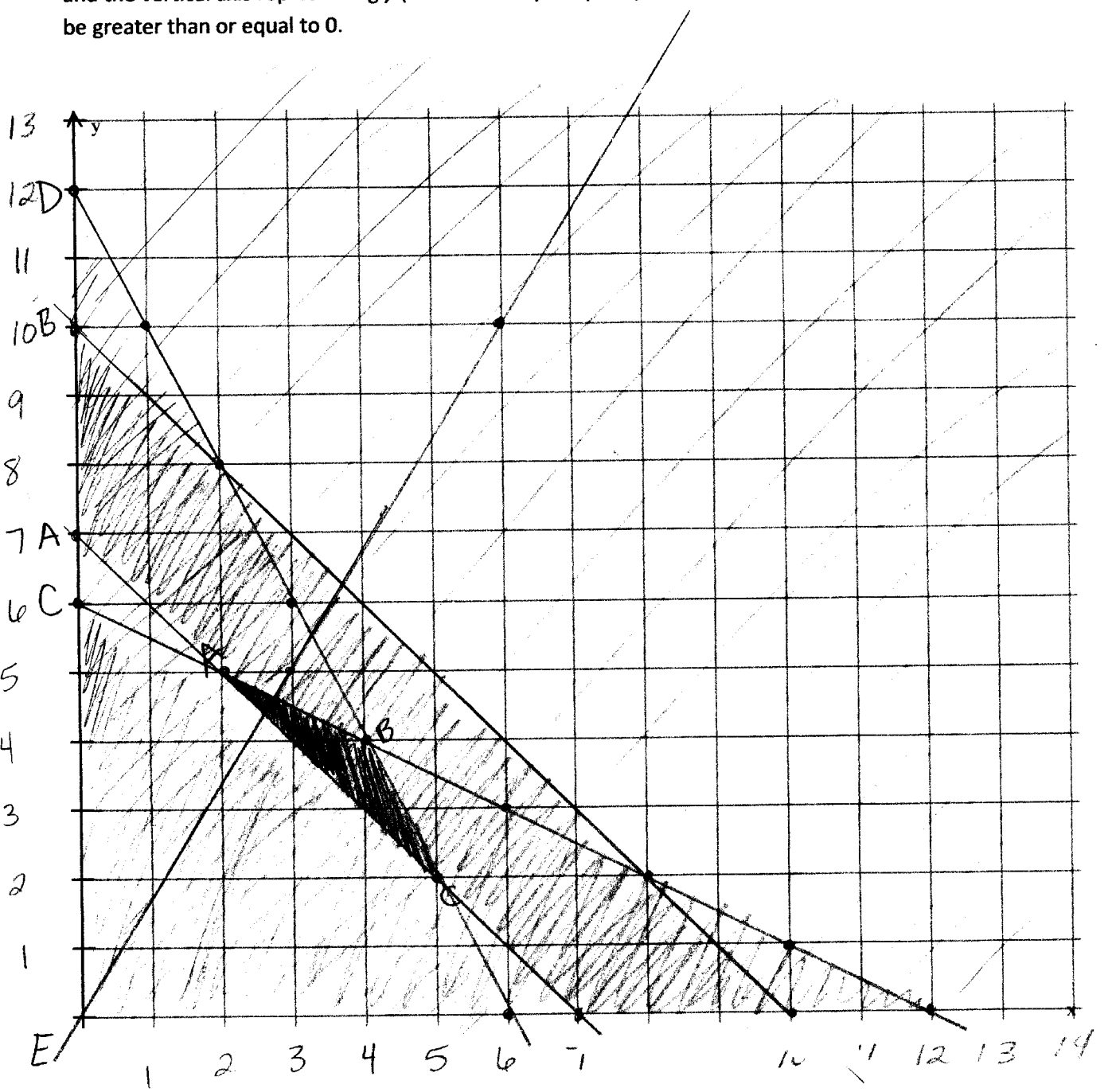
$y \geq -2x+12$

$200(0)+100(0) \leq 1200$

$\frac{2y}{2} \leq \frac{-x+12}{2}$ $y \leq -\frac{1}{2}x+6$

$0+2(0) \leq 12$

5. To solve this problem, you will need to graph the **intersection** of all five inequalities on one common XY plane. Do this on the grid below being careful to draw straight lines and to shade the correct side of each line. Let the bottom left be the origin, with the horizontal axis representing *x* (the acres of wheat to plant) and the vertical axis representing *y* (the acres of rye to plant). This is the quadrant I since both *x* and *y* must be greater than or equal to 0.



6. You should get a triangular shape where you shaded for all inequalities. Find the coordinates of the ordered pairs that make up the vertices of the triangle (the corners). To find these, you have the intersection of the two lines that make up a corner. To do this take the equations of the two lines that intersect and solve this system by substitution or elimination.

(5,2)
(4,4)
(2,5)

$x+y=7$ (1) $\quad x=7-y$

$200x+100y=1200$ (2)

$200(7-y)+100y=1200$

$1400-200y+100y=1200$

$x+2y=12 \quad x=12-2y$

$200x+100y=1200$

$200-100y=0$

$-200 \quad -200$

$\frac{-100y=-200}{-100 \quad -100}$

$y=2$

$x+2=7$

$x=5$

$200(12-2y)+100y=1200$

$2400-400y+100y=1200$

$-2400 \quad -2400$

$\frac{-300y=-1200}{-300 \quad -300}$

$y=4$

$x+2(4)=12$

$x+8=12$

$-8 \quad -8$

$x=4$

$x+y=7 \quad \cdot(-1)$

$x+2y=12$

$-x-y=-7$

$y=5$

$x+2(5)=12$

$x+10=12$

$-10 \quad -10$

$x=2$

7. You now plug each of the points you found in part 6 into the profit function to determine which ordered pair gives the maximum profit. Do this and write a sentence describing how many of each type of crop the farmer should plant to maximize his profit and stating what the maximum profit will be.

$P=500(5)+300(2)$

$P=2500+600$

$P=3,100$

$P=500(4)+300(4)$

$P=2000+1200$

$P=3200$

$P=500(2)+300(5)$

$P=1000+1500$

$P=2,500$

The farmer should plant 4 acres of wheat & 4 acres of rye to profit the maximum amount of $3,200.

Reflective Writing.

Did this project change the way you think about how math can be applied to the real world? Write one paragraph stating what ideas changed and why. If this project did not change the way you think, write how this project gave further evidence to support your existing opinion about applying math. Be specific.

When I sit down and do math, I find that I am so preoccupied with the functions and processes involved, that I forget that math is actually used in the "real world". So, when I do a project like this one it is a great tool in solidifying the concepts that I have practiced. They are no longer an abstract idea that I am trying to grasp, but rather a concrete tool that I can use to understand the world around me. Finding the maximum profit strategy in this project has shown me that there are visualizations and mathematical formulas that I can master that will help me in the future, especially when there are seemingly infinite solutions available. To be able to narrow down and determine an actual number when there was no number available to me to begin with was a great ah-ha moment. This type of process would be great to use with a variety of scenarios that could present in life such as time management, quotas, and even strategizing when playing games.